V. Fixed–Income Securities

1. Bonds and Their Pricing

- A fixed–income (bond) security is a claim on specified periodic stream of income
- A bond is easier to analyze than a stock since the level of payments of the former is fixed in advance
  - Indeed, a company is not obliged to pay dividends and even if it pays them, it may stop doing it in the future
  - Some stocks do not pay dividends (growth stocks)
  - If bond stops paying coupons, then a borrower breaks a contract and will be in state of default. The default risk for bond issuers is typically well known
- These differences will allow us to advance much more further in bond pricing than by using CAPM or APT equation
- A typical bond makes semiannual coupon payments and then pays the bond's par value (or face value) at the bond's maturity date.
  - The coupon rate is the coupon payment divided by the bond's par value
  - Bonds that do not pay coupons are called zero–coupon bonds (or pure discount bonds)
  - Bond is sold at discount (premium) if its price is smaller (bigger) than the face value
  - Corporate bonds are typically issued at par value
- Bond yield is a compensation for risk. Sources of risk in bond returns include but not limited to inflation, business cycle, and default.

- Default is more likely for corporate bonds. The cash flow becomes uncertain when default is possible.

  Question: Suppose that a bond is default–free, is it still risky?

- Price of a bond is a present value of the future cash flows that an investor expects from the bond

- Therefore, the price of a bond is a sum of the present value of a stream of coupons $C_1, C_2, ..., C_n$ and the present value of its par value:

$$P = \sum_{i=1}^{n} \frac{C_i}{(1+R)^i} + \frac{ParV}{(1+R)^n},$$

  where R is a discount rate per period.

- If all coupons are the same and equal to C then

$$P = C\frac{1-(1+R)^{-n}}{R} + \frac{ParV}{(1+R)^n}$$

- Typically, the discount rate depends on the time of coupon payment. Therefore, a more accurate formula for a bond price price is

$$P = \sum_{i=1}^{n} \frac{C_i}{(1+R_i)^i} + \frac{ParV}{(1+R_n)^n},$$

  where R_i is a discount rate per period for the payment made at period i.

  Question: How can we find the price of a given bond by using zero–coupon bonds in the market?

Practice Problem

Calculate price of a 20–year bond that makes semi–annual coupon payments. The semiannual coupon rate is 4%, the face value is $1,000, and the semiannual rate is 5% per year. Is this bond selling at a premium or a discount?

2. Bond Yields

- In practice, we observe prices and have to infer rates of return
- The average rate of return that will be earned between now and the maturity of a bond is called the **yield to maturity** (YTM)
- Yield to maturity annualized using a simple interest method is called bond equivalent yield
- Finally, yield to maturity annualized using compounded interest method is called effective annual yield
- In most practical cases the reported yield is an equivalent one. Therefore, we will follow this tradition for the rest of this topic.

Practice Problem

A 15–year bond is currently selling at par value of $1,000. Its semiannual coupon rate is 4%. Calculate its semiannual yield to maturity, equivalent annual yield, and effective annual yield.

- It follows that if bond is sold at par then $YTM = c$ (where c is a coupon rate). Moreover, if $YTM = c$ then bond is sold at par.

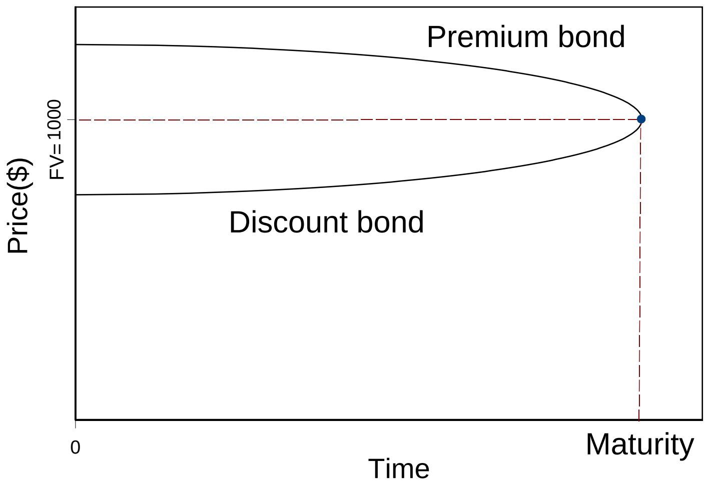
- One can show that if $YTM > c$ then bond is sold at discount and if $YTM < c$ then bond is sold at premium

- Because the yield to maturity accounts for built–in capital appreciation:

  – For a discount bond, the price will increase as the time to maturity approaches and that at the time of maturity the price will be equal to the par value

  – For a premium bond, the price will decrease as the time to maturity approaches

  – The figure below shows the above two trends



Practice Problem

Suppose that you bought a bond in the previous practice problem and held it for one year. What is your holding period return for one year? Assume that YTM does not change in 1 year and you can reinvest the first coupon payment at 4% for six months.

- The implicit assumption behind the yield to maturity is that the coupons are reinvested at the rate equal to the YTM
- If, however, coupons are not reinvested at the interest rate equal to the bond's YTM, the *realized compound yield* will not be equal to YTM
- The calculation of the realized compound yield requires calculation of actual reinvestment income
- To find the realized compound yield
  - Calculate the future value of reinvested coupons and face value at maturity day
  - Equate today's price of a bond to the present value of this bond assuming that the latter is equal to the future value above discounted at the realized compound yield.
  - Calculate the realized compound yield from the last condition.

Practice Problem

A 3–year bond that pays annual coupons of $75 currently sells for $800.00. Calculate the realized compound yield if you expect to hold the bond until maturity and to reinvest coupons at 6% per year

time	payment	future value of payment
1		
2		
3		

Question: Is realized compound yield equal to YTM in this problem?

- The interest rate for a given time interval is called the *short interest rate*
- Often, investors are interested in finding short interest rates in the future, for example between 2019 and 2020 years
- The yield to maturity on zero–coupon bond is called the *spot rate* that prevails today for a period corresponding to the maturity of the bond
- Note that short rates are per time period that could be only a part of bond duration, while the spot rate is for the whole duration of the bond
- Investors do not know short–term interest rates for coming years
- *Forward interest rates* are future short rates, inferred from the spot rates
- Suppose we are interested in the interest rate that will prevail in two years from now and we observe the following prices and yields of zero–coupon bonds

Time to maturity	Price ($)	YTM (%)
1	925.93	8.00
2	841.75	8.995
3	758.33	9.660

- Consider two strategies
  - Invest in a three–year zero–coupon bond

– Invest in a two–year zero–coupon bond. After two years, reinvest the proceeds in a one–year bond

- The forward rate is the annual rate for the third year that makes these two strategies equivalent so that the proceeds from them are the same:

$$(1 + y_3)^3 = (1 + y_2)^2(1 + f_3), \tag{5-1}$$

where y_n is the yield to maturity for year n

- Hence, the forward rate for year three is

$$f_3 = \frac{(1 + y_3)^3}{(1 + y_2)^2} - 1 = \frac{(1 + 0.0966)^3}{(1 + 0.08995)^2} - 1 = 0.11$$

- Formula (5-1) can be generalized for any period n, f_n:

$$(1 + y_n)^n = (1 + y_{n-1})^{n-1}(1 + f_n), \tag{5-2}$$

where y_n is the yield to maturity at period n.

- Clearly, the interest rate that actually will prevail in the future need not equal the forward rate

Practice Problem

Using the data above, calculate the forward rate in year 2

- Formula (5-2) can also be generalized for the forward rate for longer than 1 year time period

$$(1 + y_n)^n = (1 + y_{n-k})^{n-k}(1 + f_{n-k,n}),$$

where $f_{n-k,n}$ is a forward rate from year $n - k$ until year n from now

- If formula (5-1) is used repeatedly in its right side then one finds

$$(1 + y_n)^n = (1 + y_{n-k})^{n-k}(1 + f_{n-k+1})(1 + f_{n-k+2})...(1 + f_n) \tag{5-3}$$

- Therefore, we find

$$f_{n-k,n} = (1 + f_{n-k+1})(1 + f_{n-k+2})...(1 + f_n) - 1$$

- Note that formula (5-3) in the special case of $k = n - 1$ becomes

$$(1 + y_n)^n = (1 + y_1)(1 + f_2)(1 + f_3)...(1 + f_n)$$

Practice Problem

Find the price of 3–year zero coupon bond if one–year forward rates are

Year	annual forward rate (%)
2010	8.00
2011	9.00
2012	10.00

- The price that we expect for one–year zero–coupon bond when it is issued n years from now is

$$P = \frac{1000}{1 + r_n}$$

Therefore, we have to find the short interest rate r_n for period n

- One says that the Expectation Hypothesis holds if $f_n = r_n$ for any period in the future.

- The last hypothesis could be used to find an expected price of a bond traded in the future.

Example. Assume that the Expectation Hypothesis holds and we observe the following prices and yields of zero–coupon bonds. What should the purchase

Time to maturity	Price ($)	YTM (%)
1	925.93	8.00
2	841.75	8.995
3	758.33	9.660

price of a 2-year zero coupon bond be if it is issued at the beginning of year 2?

By definition, the price of the bond should be

$$P = \frac{1000}{1 + r_{1,3}},$$

where $r_{1,3}$ is biannual yield of the bond issued 1 year from now. According to expectation hypothesis, $r_{1,3} = f_{1,3}$, where $f_{1,3}$ is a biannual forward rate found from

$$(1 + y_3)^3 = (1 + y_1)(1 + f_{1,3}).$$

It follows from the last formula that

$$1 + f_{1,3} = \frac{925.93}{758.33} = 1.22$$

and

$$P = \frac{1000}{1 + f_{1,3}} = \frac{1000}{1.22} = \$818.99$$

- One says that the **Liquidity Preference Hypothesis** holds if $f_n = r_n + \Delta_n$ where Δ_n is the liquidity premium
- Knowing the forward rates and the liquidity premiums allows us to find expected future prices

Example. Use the previous example to find the price of a 2–year zero coupon bond issued one year from now if the Liquidity Preference Hypothesis holds and the biannual liquidity premium $\Delta_{1,3}$ is 2%.

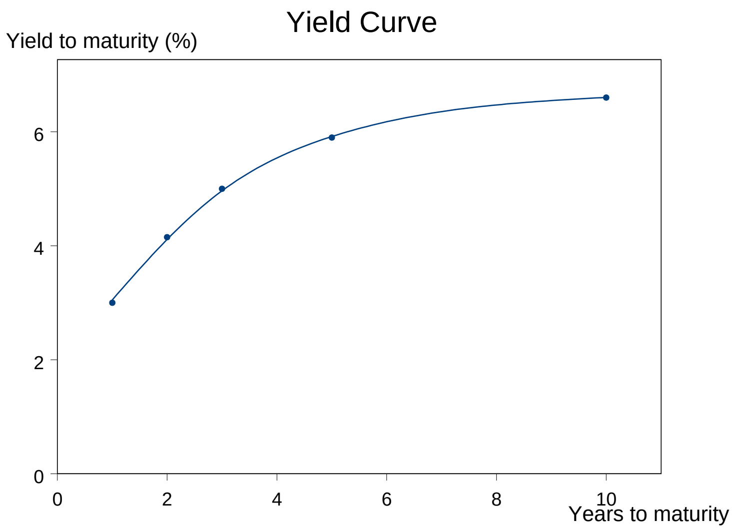
From the calculations above $1 + f_{1,3} = 1.22$ where $f_{1,3} = r_{1,3} + \Delta_{1,3}$. Therefore, $1 + r_{1,3} = 1 + f_{1,3} - \Delta_{1,3} = 1.2$ and

$$P = \frac{1000}{1 + r_{1,3}} = \frac{1000}{1.2} = \$833.33.$$

3. The Term Structure of Interest Rates

- The relationship between yield to maturity and time to maturity is called *term structure of interest rates*
- The graph that displays the relationship between yield and maturity is called the *yield curve*
- Yield curve for zero–coupon bonds provide us with all forward rates for future years
- The yield curve can be upward sloping, downward sloping or flat
- The shape of the yield curve is closely scrutinized because it helps to get an idea of future interest rate change and economic activity
  - In particular, if bond has low risk and the yield curve is upward/downward sloping, then it is expected that there will be an economic growth/recession

    Question: Why?
  - If the yield curve is flat then investors are uncertain about the future of economy
- The slope of the yield curve is also seen as important: the greater the slope, the greater the gap between short- and long-term rates
- Moreover, the slope indicates how fast an economy will switch to the state of growth or recession



4. Interest Rate Risk

- Even though the coupon payments of a bond are almost guaranteed, the interest rate may rise or fall unpredictably causing bondholders to lose or gain capitals
- Most financial institutions trade bonds and are subject to interest rate risk which is a risk of unexpected changes in interest rate
- Many of these institutions (especially pension funds, insurance companies, commercial banks) trade bonds not only for maximizing the profits but also for removing the interest rate risk
- Therefore, we discuss the interest rate risk in detail and then learn how it could be removed from portfolio

Practice Problem

Calculate price of a 20–year bond that makes semi–annual coupon payments. The annual coupon rate is 8%, the face value is $\$1,000$, and the interest rate is 8% per year. By how much would the price of the bond change if the interest rate were to (a) drop to 7%, (b) increase to 9%? What is the relationship between the level of interest rates and bond prices? Is the price of a bond more sensitive to increases or decreases in the interest rates?

- The behavior of the interest rate risk is summarized in

Six observations on the interest rate risk

1. *Bond prices and yields are inversely related*

2. *Bond prices are more sensitive to declines in a yield to maturity than to increases.* This property is known as convexity of bond prices

3. *Prices of long–term bonds tend to be more sensitive to interest rate changes than prices of short–term bonds*

Example. Consider two zero–coupon bonds that have face value of $\$1,000$ but one expires in 1 year (bond 1), while the other expires in 10 years (bond 2). What is sensitivity of the two bond prices to changes in the yield?

From the calculus,

$$\Delta P_1 = -\frac{\$1,000}{(1+y)^2}\,\Delta y,$$

$$\Delta P_2 = -10\frac{\$1,000}{(1+y)^{11}}\,\Delta y$$

Therefore,

$$\frac{\Delta P_1}{P_1} = -\frac{\Delta y}{(1+y)}, \quad \frac{\Delta P_2}{P_2} = -10\frac{\Delta y}{(1+y)}$$

Thus, the change in y results in the relative change of the bond price 10 times greater for bond 2.

4. *For bonds paying coupons, the sensitivity of their prices to changes in yields increases at decreasing rate as maturity increases*

5. *Prices of high–coupon bonds are less sensitive to changes in interest rates than prices of low–coupon bonds*

Bond sensitivity to changes in interest rate is defined by the sensitivity of the present value of coupons and the sensitivity of the present value of Par value. Since Par value is larger than coupons and is paid later, a small change in the interest rate has more significant impact on present value of the Par value than on the present value of coupons. Therefore, if coupons of a bond become larger then a small change in yield causes a smaller relative change in the price of a bond.

6. *Price sensitivity is inversely related to the yield to maturity at which the bond is selling*

This result can also be seen in our example to observation 3.

- We conclude that the interest rate risk depends not only on time to maturity, but also coupon payments and yield to maturity

- It is convenient to use an effective maturity that would take into account coupon payments and yield to maturity:

  – **Duration** (or Macaulay duration) is the weighted average of the times to each coupon or principal payment made by bond:

  $$D = \sum_{i=1}^{n} t_i \times w_i, \quad \text{where} \quad w_i = \frac{CF_i/(1+y)^i}{P},$$

  where CF_i is the payment to a bondholder at time t_i measured in years and P is the bond price today

Practise Problem

Calculate duration of a 2–year bond that makes semi–annual coupon payments. The annual coupon rate is 8%, the face value is \$1,000 and the interest rate is 10% per year.

time (years)	payment (\$)	PV of payment (\$)	w_t	$t \times w_t$
0.5	40			
1.0	40			
1.5	40			
2.0	1040			

- Bond duration allows us to find price sensitivity of the bond with fewer calculations
- Let us find the percentage change of bond price resulted from the change in y.
  - Because

    $$P = \sum_{i=1}^{n} \frac{CF_i}{(1+y)^i}$$

    we find from calculus:

    $$\Delta P = -\sum_{i=1}^{n} i \times \frac{CF_i}{(1+y)^{i+1}} \Delta y = -\frac{\Delta y}{\Delta t(1+y)} \sum_{i=1}^{n} t_i \times \frac{CF_i}{(1+y)^i},$$

    where Δt is the time between two subsequent payments (or bond period). If f is an annual frequency of payments then $f = 1/\Delta t$
  - Or, from definition of duration

    $$\Delta P = -\frac{\Delta y \times f}{(1+y)} P \times D$$

– Therefore,

$$\frac{\Delta P}{P} = -\frac{\Delta y^a}{(1+y)} D \tag{5-4}$$

where Δy^a is a change of an annual equivalent yield while y is the yield per period

– In words, proportional change in the price is equal to the proportional change in one plus the bond annual yield times the negative bond duration

– *Modified Duration* is defined as $D^\star = D/(1+y)$, so

$$\frac{\Delta P}{P} = -\Delta y^a \times D^\star$$

– $D^\star$ is the negative slope of the price–yield curve expressed as a fraction of the bond price

- Formula (5-4) allows us to estimate the change in the price resulted from a change in the yield without making cumbersome calculations. In addition, it greatly simplifies the construction of bond strategies

Practice Problem

Consider a 2–year bond that makes semi–annual coupon payments. The annual coupon rate is 8%, the face value is $1,000 and the interest rate is 10% per year. By how many percents will the price drop if the yield on the bond increases to 12%? What will be the price of the bond after such an increase in the yield?

- Note that equation (5-4) is exact only when Δy is very small. Otherwise, it is an approximation of the first order in Δy^a.

- To improve approximation of the price change, one can include the term of the second order in Δy:

$$\frac{\Delta P}{P} = -D^{\star} \times \Delta y^a + \frac{1}{2}[Convexity \times (\Delta y^a)^2],$$

where the *convexity* is the curvature of the price–yield curve expressed as a fraction of the bond price:

$$Convexity = \frac{(\Delta t)^2}{P \times (1+y)^2} \sum_{t=1}^{n} \left[\frac{CF_i}{(1+y)^i}(i^2 + i) \right],$$

where CF_i is a cash flow paid at the end of period i

- Duration allows us to quantify the interest rate sensitivity, which greatly enhance our ability to formulate investment strategies

- Therefore, it is important to understand the determinants of duration

Rules for Duration

Rule 1. Holding maturity and yield constant, a bond's duration is higher when the coupon rate is lower

Indeed, the lower coupon rate, the lower the weight of these coupons in duration. Thus, the weight of the principal increases and so does duration

Rule 2. Holding the coupon rate and yield constant, a bond's duration generally increases with its time to maturity

As time to maturity increases, the present value of the principal as well as the late coupons will decrease. Generally this decrease is outweighed by increase in the times of payments, so duration rises

If yield to maturity is high, then duration may fall with increasing maturity

Rule 3. Holding other factors constant, the duration of a coupon bond is higher when the bond's yield to maturity is lower

If yield decreases then the weight of Par value in duration increases. Therefore, duration goes up

Rule 4. The duration of a zero–coupon bond equals its time to maturity

This rule is easy to verify from definition of duration

Rule 5. The duration of a level perpetuity is

$$\frac{1+y}{y}\Delta t$$

Indeed, since the price of a level perpetuity is $P = \frac{C}{y}$, we find from calculus

$$\Delta P = -\frac{C}{y^2}\Delta y$$

and

$$\frac{\Delta P}{P} = -\frac{1}{y}\Delta y.$$

The result follows from solving the following equation for D:

$$\frac{\Delta P}{P} = -\frac{D}{1+y}\frac{\Delta y}{\Delta t} = -\frac{1}{y}\Delta y.$$

Rule 6. The duration of a level annuity is

$$\Delta t\left(\frac{1+y}{y} - \frac{n}{(1+y)^n - 1}\right)$$

Note, that level annuity is a bond that pays coupons at time intervals Δt for n times and does not pay the principal

Rule 7. The duration for a corporate bond is

$$\Delta t\left(\frac{1+y}{y} - \frac{(1+y) + n(c-y)}{c[(1+y)^n - 1] + y}\right),$$

where c is a coupon rate

5. Managing Bond Portfolios

- There are two main types of bond management: passive and active

  Passive strategies

- Take bond prices as fairly set and seek to control only the risk
- There are two major classes of passive strategies
  - Bond index portfolio.

    This is an indexing strategy that attempts to replicate the performance of a given bond index (Scotia Capital Universe Index in Canada)

    Difficulties:

    * Too many bonds
    * Some bonds are very illiquid
    * Maturities of many bonds are relatively short, so a manager must constantly update her portfolio
  - Immunization.

    It is designed to shield from the interest rate fluctuations.

    * It is used by financial institutions, insurance companies, and pension funds
    * For example, in the case of banks, deposits (liabilities) are usually short–term while loans (assets) are usually long–term. The sensitivity to the interest rate fluctuations are higher for loans than for deposits. If interest rates increase unexpectedly then the value of bank loans drops more significantly than that of deposits making banks loose money
    * To eliminate the gap between the sensitivities of deposits and loans, banks engage in *net worth immunization*, which is achieved through equating the products of liabilities by their duration and assets by their duration ($L \times D_L = A \times D_A$)

Example

Suppose you are a pension fund manager and one of your clients has just made a $\$10,000$ deposit in a GIC (GICs are zero–coupon bonds issued by company) with a maturity of 5 years. The current rate of the 5–year GICs is 8% per year.

5 years from now you will have to pay $(1+0.08)^5 \times \$10,000 = \$14,693.28$ and the duration of your obligation is 5 years.

You could fund the obligation with $\$10,000$ of 8 percent annual coupon bond, selling at par, with 6 years to maturity. You will sell this bond in 5 years hoping to cover your liabilities

- The duration of your bond (asset) is

$$D = \frac{1+y}{y} - \frac{(1+y)+n(c-y)}{c[(1+y)^n - 1] + y} = \frac{1+0.08}{0.08} - \frac{(1+0.08)+6(0.08-0.08)}{0.08[(1+0.08)^6 - 1] + 0.08} = 5$$

- If the interest rate does not change then the accumulated funds will grow to exactly $\$14,693.28$. However, it is very likely that the rate is going to change

- The table below shows the dynamics of assets and liabilities for the case when the interest rate remains at 8% and when it drops to 7% right after the GIC is sold.

  Note that you face two off–setting types of risk:

  * Price risk. (If interest rate rise, the fund will suffer the capital loss. The bonds will be worth less in 5 years than if the rates had remained at 8%)

  * Reinvestment risk. (However, at higher interest rate, reinvested coupons will grow at a faster rate, offsetting the capital loss)

- Two risks offset each other almost completely because duration of your assets is equal to the duration of liabilities

Payment Number	Years Remaining Until Obligation	Accumulated Value of Invested Payment	
Rates remain at 8%			
1	4	$800 \times (1.08)^4$	=1,088.39
2	3	$800 \times (1.08)^3$	=1,007.77
3	2	$800 \times (1.08)^2$	=933.12
4	1	$800 \times (1.08)^1$	=864.00
5	0	$800 \times (1.08)^0$	=800.00
Sale of bond	0	10,800/1.08	=10,000.00
			14,693.28
Rates fall to 7%			
1	4	$800 \times (1.07)^4$	=1,048.64
2	3	$800 \times (1.07)^3$	=980.03
3	2	$800 \times (1.07)^2$	=915.92
4	1	$800 \times (1.07)^1$	=856.00
5	0	$800 \times (1.07)^0$	=800.00
Sale of bond	0	10,800/1.07	=10,093.46
			14,694.05

  - Our assets will not be exactly equal to our liabilities because duration allows to remove the risk only approximately. To improve risk management, one can take into account convexity or try to rebalance immunized portfolio more often.

Practice Problem

Finish the table above assuming that the yield were to rise to 10% right after you bought the bond.

Active strategies

- There are at least two sources of potential value in active trading: (1) interest

Payment Number	Years Remaining Until Obligation	Accumulated Value of Invested Payment	
Rates rise to 10%			
1	4		
2	3		
3	2		
4	1		
5	0		
Sale of bond	0		____________

rate forecasting and (2) identification of relative mispricing

- These techniques will generate abnormal returns only if the analyst's insight is superior to that of the market
- Interest rate forecasting include at least one active strategy
  - The *rate anticipation swap* is based on superior information that the rate will rise or fall. For example, if it will fall, an investor will swap into bonds of longer duration and enjoy a greater percentage increase in price
- Finding inefficiencies includes at least two active strategies
  - The *substitution swap* is an exchange of one bond for nearly identical substitute. The bonds should be of essentially equal coupon, maturity, quality, call features, and so on, but have significantly different yields.
  - The *intermarket spread swap* is pursued when investor believes that the yield spread between two sectors of the bond market (e.g., corporate and government bonds) is temporally out of line

Additional Practice Problems

1. A coupon bond that pays interest of \$100 annually has a par value of \$1,000, matures in 5 years, and is selling today at a \$72 discount from par value. The yield to maturity on this bond is

A) 6.00 %

B) 8.33 %

C) 12.00 %

D) 60.00 %

E) none of the above

2. You purchased an annual interest coupon bond one year ago that had 6 years remaining to maturity at that time. The coupon interest rate was 10% and the par value was \$1,000. At the time you purchased the bond, the yield to maturity was 8%. If you sold the bond after receiving the first interest payment and the yield to maturity continued to be 8%, your annual total rate of return on holding the bond for that year would have been

A) 7.00 %

B) 7.82 %

C) 8.00 %

D) 11.95 %

E) none of the above

Use the following to answer questions 3-18:

The following is a list of prices for zero coupon bonds with different maturities and par value of \$1,000.

Maturity (Years)	Price ($)
1	943.40
2	881.68
3	808.88
4	742.09

3. What is, according to the expectations theory, the expected forward rate in the third year?

A) 7.00 %

B) 7.33 %

C) 9.00 %

D) 11.19 %

E) none of the above

4. What is the yield to maturity on a 3-year zero coupon bond?

A) 6.37 %

B) 9.00 %

C) 7.33 %

D) 10.00 %

E) none of the above

5. What is the price of a 4-year maturity bond with a 12% coupon rate paid annually? (Par value = $\$1,000$)

A) $\$742.09$

B) $\$1,222.09$

C) $\$1,000.00$

D) $\$1,141.84$

E) none of the above

6. Consider two annual coupon bonds, each with two years to maturity. Bond A has a 7% coupon and a price of $\$1,000.62$. Bond B has a 10% coupon and sells for $\$1,055.12$. Find the two one-period forward rates that must hold for these bonds.

A) 6.97%, 6.95%

B) 6.95%, 6.95%

C) 6.97%, 6.97%

D) 6.08%, 7.92%

E) 7.00%, 10.00%

7. Holding other factors constant, which one of the following bonds has the smallest price volatility?

A) 5-year, 0% coupon bond

B) 5-year, 12% coupon bond

C) 5 year, 14% coupon bond

D) 5-year, 10% coupon bond

E) Cannot tell from the information given.

8. The duration of a par value bond with a coupon rate of 8% and a remaining time to maturity of 5 years is

A) 5 years.

B) 5.4 years.

C) 4.17 years.

D) 4.31 years.

E) none of the above.

9. Which one of the following par value 12% coupon bonds experiences a price change of \$23 when the market yield changes by 50 basis points?

A) The bond with a duration of 6 years.

B) The bond with a duration of 5 years.

C) The bond with a duration of 2.7 years.

D) The bond with a duration of 5.15 years.

E) None of the above.

10. You have an obligation to pay $\$1,488$ in four years and 2 months. In which bond would you invest your $\$1,000$ to accumulate this amount, with relative certainty, even if the yield on the bond declines to 9.5% immediately after you purchase the bond?

A) a 6-year; 10% coupon par value bond

B) a 5-year; 10% coupon par value bond

C) a 5-year; zero-coupon bond

D) a 4-year; 10% coupon par value bond

E) none of the above